A new result from the Creature World

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joint work with Lukas Klausner and Diego Mejía

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Set Theory: Reals and topology

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Each $\mathcal{I}_f$ is a σ-ideal and $\mathcal{SN} \subseteq \mathcal{I}_f \subseteq \mathcal{N}$.

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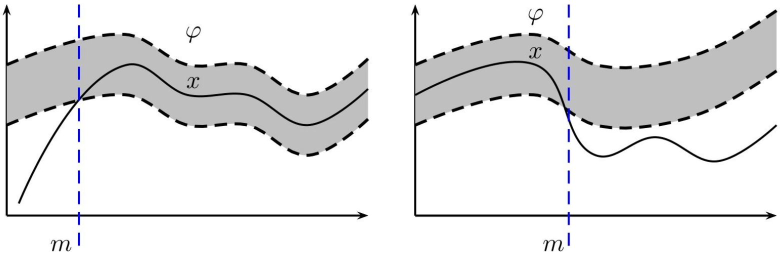

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2. $\mathfrak{d}^{\mathrm{aLc}}_{c_\kappa,h_\kappa} = \mathrm{non}(\mathcal{I}_{f_\kappa}) = \mathfrak{d}^{\mathrm{Lc}}_{a_\kappa,d_\kappa} = \kappa$, for all $\kappa \in A$.

Questions

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2. $\mathfrak{b}^{\mathrm{Lc}}_{b,h}$ (without large cardinals)?

Main result

Main theorem (C., Klausner and Mejía)

Assume CH. Let μ be an infinite cardinal such that $\mu = \mu^{\aleph_0}$ and let $\{\kappa_i : i \in S^{\mathrm{sk}}\}$ be a set of infinite cardinals below μ such that $|S^{\mathrm{sk}}| = \mu$ and $\kappa_i^{\aleph_0} = \kappa_i$ for all $i \in S^{\mathrm{sk}}$.

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(i) $b(n)/g(n) > d(n)$ for all but finitely many $n \in \omega$.

(ii) $a(n) \geq |[b(n)]^{\leq g(n)} \smallsetminus \{\varnothing\}|$ for all but finitely many $n \in \omega$.

(iii) $\langle J_n : n \in \omega\rangle$ is a interval partition of ω, $|J_n| = g(n)$.

(iv) $f_{b,g}(k) := \sum_{i \leq n} \lceil \log_2 b(i) \rceil$ when $k \in J_n$.

(v) f is increasing and $f_{b,g}(k) \leq^* f \circ \mathrm{id}^m$ for some $m \geq 1$.

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Lemma (Klausner and Mejía 2018)

If $\vec{\nu}$ is a block then $\mathfrak{d}^{\mathrm{aLc}}_{c,h} \leq \mathrm{non}(\mathcal{I}_f) \leq \mathfrak{d}^{\mathrm{aLc}}_{b,g} \leq \mathfrak{d}^{\mathrm{Lc}}_{a,d}$.

Corollary

Any block satisfy too that,

$$\mathfrak{b}^{\mathrm{Lc}}_{a,d} \leq \mathfrak{b}^{\mathrm{aLc}}_{b,g} \leq \mathrm{cov}(\mathcal{I}_f) \leq \mathfrak{b}^{\mathrm{aLc}}_{c,h} \leq \mathfrak{d}^{\mathrm{Lc}}_{c,h}$$

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Say that two blocks $\vec{\nu}$ and $\vec{\nu}'$ are *mutually independent* if

$$\lim_k \left(\min \left\{ \frac{[c(k)]^{\leq h(k)}}{d'(k)}, \frac{a'(k)}{d(k)} \right\} \right) = 0$$

for any $k \in \omega$.

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Motivation I

Fix functions $h, c, d \in \omega^\omega$ such that

(i) $1 \leq h < c$,

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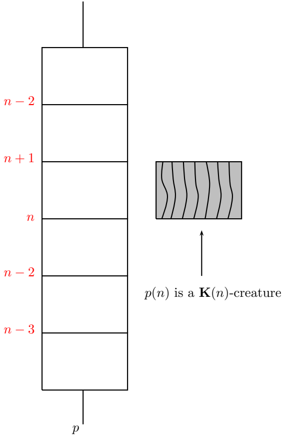


Klausner and Mejía 2018

The lim-sup forcing $\mathbb{P}$ forces $(\star)$

Klausner and Mejía 2018

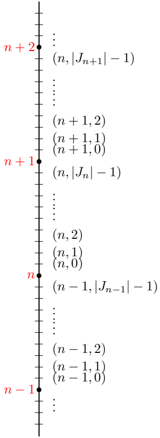
The lim-sup forcing $\mathbb{P}$ forces $(\star)$ whenever:

(i) $\forall^\infty k : \prod_{i<k} a'(k) \leq d(k), \prod_{i<k} c(k)^{h(k)} \leq d'(k)$, and

(ii) $\lim_k \left(\min \left\{ \frac{c(k)^{h(k)}}{d'(k)}, \frac{a'(k)}{d(k)} \right\} \right) = 0$

Motivation II

For each $n \in \omega$ we fix some J_n with $0 < |J_n| < \omega$.



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Let $\Delta := \{(n,u) : n \in \omega, u \in J_n\}$ and let $\hat{a}, \hat{d} : \Delta \to \omega$.
Fix functions $a, d \in \omega^\omega$ such that

$a(n) = \prod_{u \in J_n} \hat{a}(n,u)$

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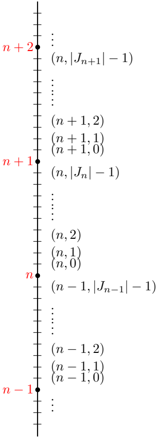

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- We want to find a forcing notion $\mathbb{P}$ increasing $\mathfrak{b}^{\mathrm{Lc}}_{a,d}$, i.e. adds a generic slalom $\varphi \in \mathcal{S}(a,d)$ localizating all ground model reals, that is,

$$(\blacktriangle)\ \ \forall x \in \prod a \cap V (x \in^* \varphi).$$

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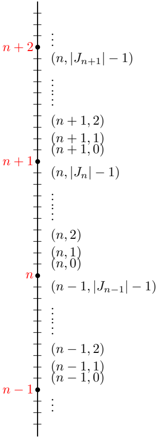

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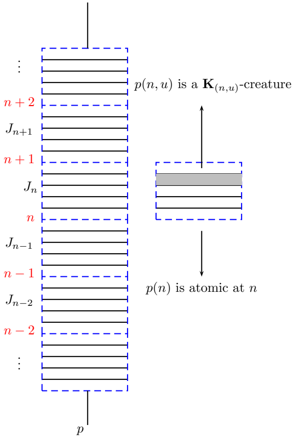

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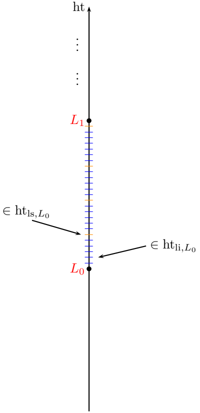

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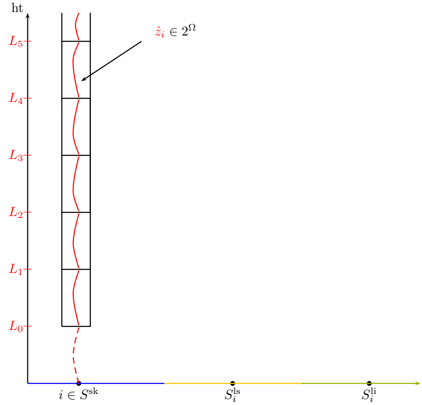

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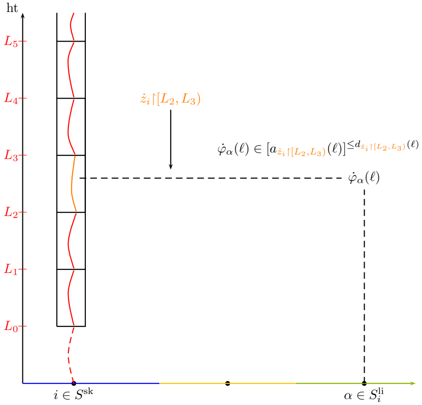
For example, for $\ell \in \mathrm{ht}_{\mathrm{li},L_2}$ we have that $z_i{\restriction}[L_2, L_3)$ determines a fragment of the block $\vec{\nu}_{z_i}$



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  - $\mathbf{x} = \langle \mathbf{x}(t) : t \in 2^J \rangle$ and
  - $\mathbf{x}(t)$ is a $\mathbf{K}_{\ell,t}$-creature.

(ii) An *atomic creature* $\mathbf{x}$ along J consists of a sequence $\langle \mathbf{x}(\ell) \rangle_{\ell \in J}$ where $\mathbf{x}(\ell)$ is a $\mathbf{K}_\ell$-creature, that is,
  - $\mathbf{x}(\ell) = \langle \mathbf{x}(\ell, t) : t \in 2^J \rangle$ and

Simple creature systems at ℓ

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Open question

Is it consistent that $\mathrm{add}(\mathcal{I}_f) = \mathrm{add}(\mathcal{I}_g)$ for **some** f, g?
Same problem with cof.