Set Theory: Reals and Topology

集合論：連続体上の組合せ論と位相空間論

November 16th (Mon.)～ 20th (Fri.), 2020

Last update: November 12th, 2020

RIMS Workshop 2020

Set Theory: Reals and Topology

Program

Time in Japan Standard Time (GMT +9)

Mon.	Opening		Guzmán (Mini course)				Parente				
Nov. 16th	9:00-9:10		9:10-10:10				10:40-11:20				
Tue.	Martínez-Ranero			Ishiu			Lambie-Hanson				
Nov. 17th	9:00-9:30			9:45-10:15			10:30-11:00				
Wed.	Guzmán (Mini course)						Jirattikansakul		Yoshinobu		
Nov. 18th	9:00-10:00						10:30-11:00		11:15-11:35		
Thu.	Ramos-García (Invited)						Sakai				
Nov. 19th	9:00-10:00						10:30-11:10				
Fri.	Guzmán (Mini course)						Sonpanow				
Nov. 20th	9:00-10:00						10:30-11:00				

Kumar (Mini course)						Schilhan				Fischer	
14:00-15:00						15:25-15:55				16:10-16:50	
Kumar (Mini course)						Eskew				Agostini	
14:00-15:00						15:25-15:55				16:10-16:50	

Kumar (Mini course)						Cardona				Supina	
14:00-15:00						15:25-15:55				16:10-16:50	
Fuchino			Illanela				Goldstern (Invited)				
14:00-14:40			15:05-15:35				15:50-16:50				

1 Tutorials

Osvaldo Guzmán

Some Constructions of MAD families

A MAD family A is called completely separable if every set that is not in the ideal generated by A, contains an element of A. A famous problem of Erdos and Shelah asks if such MAD family can be constructed in ZFC alone. In his 935 paper, Shelah developed a novel method to construct MAD families with special properties. In that paper, he constructed a completely MAD family under different assumptions (providing an "almost" ZFC result). This method has been simplified and expanded by Mildenberger, Raghavan, and Steprans. In this tutorial, we will explain this method to construct special MAD families and some of its applications.

Ashutosh Kumar

Forcing with sigma ideals and Lebesgue outer measure

We will discuss some applications of forcing and generic ultrapowers to Ramsey-type problems about Lebesgue outer measure.

2 Invited talks

Martin Goldstern

Many cardinals below the continuum

It is well known that "the continuum can be anything it ought to be", and in particular, that the interval from $\aleph_1$ to $\mathfrak{c} = 2^{\aleph_0}$ can contain arbitrarily many cardinals. It is also an old result that there can be many cardinals in this interval which all have particularly simple definitions.

I my talk I will sketch the construction of a ZFC model where many of the well known cardinal characteristics of the continuum (in particular, the cardinals from Cichoń's diagram, and some others such as $\mathfrak{p}$ and $\mathfrak{s}$) can be "as distinct as they ought to be".

The construction uses a finite support iteration of small forcing notions. The most recent element of the construction (which is responsible for the value of the splitting number $\mathfrak{s}$) exploits the homogeneity of the forcings we use.

This is joint work with Jakob Kellner, Diego Mejía and Saharon Shelah.

Ulises Ariet Ramos-García

OCA in the class of hereditarily Lindelöf submetrizable spaces

The purpose of this talk is to analyze Todorcevic's Open Coloring Axiom (OCA) in the class of hereditary Lindelöf submetrizable spaces. This class naturally extends to the class of all separable metrizable spaces where OCA is typically defined. Among other results, we show that the definable version for OCA (OCA*) holds for the class of all hereditary Lindelöf strong Choquet submetrizable spaces. In particular, OCA* holds for the Sorgenfrey line and for the space $(\omega^\omega, \tau_{GH})$ where τ_{GH} is the Gandy-Harrington topology on ω^ω.

This is joint work with José Antonio Corona-García.

3 Contributed talks

Claudio Agostini

A characterization of finite Ramsey monoids

Joint work with Eugenio Colla.

Carlson's theorem on variable words and Gowers' FIN_K theorem are generalizations of Hindman's theorem that involve a monoid action on a semigroup. In short, they state that for any finite coloring of a semigroup there is an infinite monochromatic "span". They differ in the choice of the monoid.

Recently, Solecki isolated from these two theorems the notion of Ramsey monoid, providing a common generalization of them. Then he proved that an entire class of finite monoids is Ramsey.

In this talk, I will present some of the result from a joint work with Eugenio Colla, where we prove a generalization of Solecki's theorem, enlarging the class of monoids that can be proved to be Ramsey and reaching a simple algebraic characterization of finite Ramsey monoids.

Miguel A. Cardona

A new result from the creature world

In this talk we use a countable support construction of creature posets to force that there are continuum many pairwise different cardinal invariants where continuum many are of each of the following types: covering numbers associated with Yorioka ideals; uniformity numbers associated with a Yorioka ideals; localization cardinals; and anti-localization cardinals.

This is a joint work with Diego Mejía and Lukas D. Klausner.

Monroe Eskew

Weak square from weak presaturation

Can we have both a saturated ideal and the tree property on $\aleph_2$? Towards the negative direction, we show that for a regular cardinal κ, if $2^{<\kappa} \leq \kappa^+$ and there is a weakly presaturated ideal on κ^+ concentrating on cofinality κ, then $\square^*_\kappa$ holds. This partially answers a question of Foreman and Magidor about the approachability ideal on $\aleph_2$. A surprising corollary is that if there is a weakly presaturated ideal J on $\aleph_2$ such that $P(\aleph_2)/J$ is a proper forcing, then CH holds. This is joint work with Sean Cox.

Vera Fischer

The spectrum of independence

In this talk, we will show that the set of cardinalities of maximal independent families, referred to as the spectrum of independence, can be quite arbitrary. In particular, we will show that every infinite subset of $\{\aleph_n\}_{1\geq n\in\omega}$ can be realised (in a generic extension) as the spectrum of independence. This is a joint work with S. Shelah.

Sakae Fuchino

Rado's Conjecture and Hamburger's Hypothesis

Hamburger's Hypothesis $\mathsf{HH}(<2^{\aleph_0})$ with the reflection point "$<2^{\aleph_0}$" is known to be independent over a combination of other strong reflection principles where the combination includes Rado's Conjecture $\mathsf{RC}(\leq 2^{\aleph_0})$ with the reflection point "$\leq 2^{\aleph_0}$" as well as the $\mathcal{P}_\kappa(\lambda)$ version of the stationarity reflection with the reflection point $<2^{\aleph_0}$ which implies that the continuum is fairly large. (see [1]).

In this talk, we show that we can refine the method in [1] to show that $\mathsf{HH}(<2^{\aleph_0})$ is independent over other reflection principles including $\mathsf{RC}(<2^{\aleph_0})$.

We also show that Rado's Conjecture RC (with the reflection point $\leq\aleph_1$) does not imply Hamburger's Hypothesis HH (with the reflection point $\leq\aleph_1$) whose consistency is a persistently open problem.

[1] Sakaé Fuchino, André Ottenbreit Maschio Rodrigues and Hiroshi Sakai, *Strong downward Löwenheim-Skolem theorems for stationary logics, III — mixed support iteration*, to appear in the ***Proceedings of the Asian Logic Conference*** 2019.
Extended version of the paper:
https://fuchino.ddo.jp/papers/SDLS-III-xx.pdf

Martina Iannella

The complexity of convex bi-embeddability between countable linear orders and some applications

Consider the set LO of countable linear orders and the following relation of "convex embeddability" among them:

$$L \trianglelefteq_{LO} M \text{ iff } L \text{ is isomorphic to a convex set in } M.$$

One easily gets that $\trianglelefteq_{LO}$ is an analytic quasi-order on the Polish space LO. We show that, in contrast to the usual embeddability between linear orders, the relation $\trianglelefteq_{LO}$ is combinatorially complicated: it is not a well quasi-order (wqo). Indeed, it has both innite descending chains and antichains of size continuum. Denote by $\bowtie_{LO}$ the equivalence relation on LO induced by $\trianglelefteq_{LO}$.

Theorem. *(i) The isomorphism relation $\cong_{LO}$ between linear orders is Borel reducible to $\bowtie_{LO}$. In particular, $\bowtie_{LO}$ is a proper analytic equivalence relation.*

(ii) $\bowtie_{LO}$ is not complete for analytic equivalence relations.

Moreover, we have evidence that $\bowtie_{LO}$ is not much more complicated than $\cong_{LO}$. Indeed, we found a partition of LO in four subsets such that the restriction of $\bowtie_{LO}$ on each of them is Borel reducible to $\cong_{LO}$.

As an application, after reviewing a recent result of Kulikov on the classification of wild proper arcs/knots up to equivalence [1], we consider a binary relation called "component" which is crucial to that proof. Since such relation turns out to be a quasi-order, we consider the problem of classifying (wild) arcs up to the associated equivalence relation and we show that the complexity of such relation is bounded from below by $\bowtie_{LO}$.

[1] V. Kulikov, *A Non-classification Result for Wild Knots*, ***Transactions of the American Mathematical Society***, vol. 369 (2017) n. 8, pp. 5829–5853.

Tetsuya Ishiu

There is no Peano Curve for a compact connected nowhere separable linearly order topological space

We shall show that if L is a compact connected nowhere separable linearly ordered topological space, then there is no continuous surjection from L onto the product of two copies of L. It means that the existence of space-filling curves such as the Peano Curve is specific to the real interval.

Sittinon Jirattikansakul

Blowing up singular cardinal of uncountable cofinality with collapses

One of the main combinatorial principles in set theory is singular cardinal hypothesis (SCH). It was known by Gitik that to violate SCH, large cardinal assumption is required. Recently Gitik built up a forcing to violate SCH at any given cofinality, assuming the cardinal is singular in the ground model. In this talk, I will present a similar forcing with collapses to violate SCH at small cardinals with the same setting.

Chris Lambie-Hanson

Highly connected Ramsey theory on the real numbers

In recent work, Bergfalk, Hrusak, and Shelah introduce some natural nontrivial weakenings of the classical partition relation $\mu \to (\mu)^2_\kappa$ that can consistently hold for uncountable but non-weakly compact cardinals μ. Roughly speaking, these weakenings replace the assertion of the existence of large monochromatic complete subgraphs in the classical partition relation with the assertion of the existence of large highly connected monochromatic subgraphs. We present some recent work on this topic, including some results about the influence of PFA and square principles on these partition relations. Our main result is a sharp consistency result indicating that a square-bracket version of this partition relation can consistently hold at the continuum, and that this is in fact equiconsistent with the existence of a weakly compact cardinal. Time permitting, we will discuss some potential higher-dimensional generalizations of these results.

Carlos Martínez-Ranero

Hereditary interval algebras and cardinal characteristics of the continuum

An interval algebra is a Boolean algebra which is isomorphic to the algebra of finite unions of half-open intervals, of a linearly ordered set. An interval algebra is hereditary if every subalgebra is an interval algebra. We answer a question of M. Bekkali and S. Todorevi, by showing that it is consistent that every σ-centered interval algebra of size $\mathfrak{b}$ is hereditary. We also show that there is, in ZFC, an hereditary interval algebra of cardinality $\aleph_1$.

Francesco Parente

Combinatorics of ultrafilters on Boolean algebras

I will present recent joint work with Jörg Brendle, concerning the structure of ultrafilters on Boolean algebras in the framework of Tukey reducibility. The main motivation is to understand and classify (not necessarily generic) ultrafilters on the algebras adjoining

Cohen and random reals. Furthermore, I will highlight the connection with a cardinal invariant of Boolean algebras, the ultrafilter number, and prove consistency results concerning its possible values.

Hiroshi Sakai

Higher stationary reflection and cardinal arithmetic

The Weak Reflection Principle, WRP, is known to have many interesting consequences. Among other things, it implies that $2^\omega \leq \omega_2$ and the Singular Cardinal Hypothesis holds. In this talk, we discuss higher analogs of WRP. We study their consequences on cardinal arithmetic.

Nattapon Sonpanow

Cardinal characteristics associated with families of functions and permutations

A cardinal characteristic of the continuum is a cardinal that lies between $\aleph_1$ and $\mathfrak{c}$ which describes a combinatorial property of the continuum. These cardinals are mostly defined on families of infinite sets of natural numbers. We study cardinal characteristics associated with families of functions and permutations on the set of natural numbers with some combinatorial properties. We give relations among these cardinals and other well-known ones.

Jonathan Schilhan

Definability of maximal families of reals in forcing extensions

We study the definability of maximal independent sets in analytic hypergraphs. This includes many of the classical types of "maximal families of reals", such as mad families, ultrafilters, maximal indpendent families, Hamel bases or Vitali sets. Our main result is that, after a countable support iteration of Sacks forcing or of splitting forcing (a less known forcing adding splitting reals) over L, every analytic hypergraph on a Polish space has a $\mathbf{\Delta}^1_2$ maximal independent set. This means that in the models obtained by these iterations, most types of interesting "maximal families" have $\mathbf{\Delta}^1_2$ witnesses. As a corollary, this solves an open problem of Brendle, Fischer and Khomskii by providing a model with a Π^1_1 mif (maximal independent family) while the independence number $\mathfrak{i}$ is bigger than $\aleph_1$.

Jaroslav Supina

Ideal Fréchet–Urysohn property of a space of continuous functions

Let us recall a folklore convergence concept by H. Cartan from 1937, stated in terms of ideals, for set of reals X and associated space $\mathrm{C}_p(X)$, the space of real-valued continuous functions on X with topology of pointwise convergence. For a continuous function f in the closure of $A \subseteq \mathrm{C}_p(X)$ there is a sequence $\{f_n \colon n \in \omega\} \subseteq A$ and an ideal $\mathcal{I}$ on natural numbers such that $\langle f_n \colon n \in \omega \rangle$ converges to f with respect to $\mathcal{I}$.

We shall focus on sets of reals X such that for a fixed ideal $\mathcal{I}$ and any continuous f in the closure of $A \subseteq \mathrm{C}_p(X)$ there is a sequence $\{f_n \colon n \in \omega\} \subseteq A$ converging to f with respect to $\mathcal{I}$.

Some of the presented results were obtained with S. Bardyla, V. Šottová or L. Zdomskyy.

Yasuo Yoshinobu

Fragility of properness

We prove that for any models $V \subseteq W$ of ZFC with the same ordinals, there is a poset which is proper in V but not in W. This answers a question raised by Karagila.